

## **CPM – COLLEGE PREPARATORY MATH** **(6<sup>th</sup> through 12<sup>th</sup> Grade)**

The Twin Valley School District uses College Preparatory Mathematics (CPM) in the middle grades (6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup>) and in our secondary math program (9<sup>th</sup> thru 12<sup>th</sup> grade). CPM was designated one of five “Exemplary Mathematics Programs” in the country by the U.S. Department of Education in October 1999. The curriculum emphasizes a variety of teaching methodologies including lecture, class discussions, and structured study teams. During class, students are actively working on guided investigations, math labs, to develop mathematical concepts and problem solving skills.

The goals for CPM are:

- To help more students learn more about mathematics effectively
- Regenerate student interest and performance in college preparatory mathematics
- Enable students to take more college preparatory mathematics classes
- Provide students with a mathematics education that will make them competitive and successful in the marketplace.

The courses in 6<sup>th</sup> and 7<sup>th</sup> grade, Foundations for Algebra Year 1 (FFA 1) and Foundations for Algebra Year 2 (FFA 2), are designed to prepare students for Algebra 1 in 8<sup>th</sup> grade. Students then take Geometry in 9<sup>th</sup> grade, Algebra 2 in 10<sup>th</sup> grade, Pre-calculus or math analysis in 11<sup>th</sup> grade and Calculus in 12<sup>th</sup> grade.

The following is a list of each of the math courses and their specific content. You can also visit CPM’s website at [www.cpm.org](http://www.cpm.org) .

## FOUNDATIONS FOR ALGEBRA YEAR 1 (FFA 1)—THE BIG PICTURE

- Concepts are based on memorable activities or concrete models so more students will be successful. For example the integer arithmetic is introduced with a concrete model, integer tiles, and practiced in game formats before moving to the abstract.
- The first five chapters cover:
  - Addition, multiplication and subtraction of integers.
  - Central tendency, mean, mode, median, and stem-and-leaf plots.
  - Setting up a complete graph and scaling axes.
  - Coordinate graphing.
  - Variables and solving two step algebraic equations.
  - Absolute value.
  - Using Guess and Check to solve word problems.
  - Fraction, decimal, percent conversion.
  - Simplifying expressions using the order of operations.
  - Basic geometric vocabulary.
  - Area of rectangles, parallelograms, triangles, and trapezoids.
  - Geometric model for multiplication.
  - Use of subproblems to solve more complex area problems.
  - Equivalent fractions using fraction bars, rulers, grids, and ratio tables.
  - Identity Property of Multiplication (seen as a “Giant 1”).
  - Solving proportions, unit costs, and unit rates using a ratio tables.
- Chapters six through 10 cover:
  - Setting up proportions and solving using cross multiplication in real life applications including currency conversion, distance rate and time, percent, sale price, discount, similar figure, and scale drawings.
  - Least common multiples.
  - Changing fractions to decimals using long division.
  - Calculate probability of complementary events.
  - Division of integers.
  - Fraction and decimal arithmetic.
  - Geometric angle concepts.
  - Classifying triangles.
  - Combining like terms.
  - Number properties including the Distributive property, commutative property, associative Property and Identity Property.
  - Circumference and area of circles.
  - Volumes of prisms and cylinders.
  - Surface area.
  - Experimental and theoretical probability.
  - Sampling techniques, bias in polls, analyzing the validity of claims, and correlation versus causation.
- Each chapter includes mental math and reviews the concepts developed previously. All chapters contain a culminating big problem or summary activity to tie together and/or possibly extend the concepts.

## FOUNDATIONS FOR ALGEBRA YEAR 2 (FFA 2)—THE BIG PICTURE

- Concepts are based on memorable activities or concrete models so more students will be successful. For example multiplication and factoring of polynomials are introduced with a concrete model, algebra tiles, before moving to the abstract.
- The first five chapters cover:
  - Data display and interpretation using scatter plots, line graphs, bar graphs, stem-and-leaf plots, and box-and-whisker plots.
  - Guess and Check tables solve word problems.
  - Measures of central tendency.
  - Graphing ordered pairs, lines and parabolas.
  - Patterns and rules in tables.
  - Arithmetic of integers.
  - Probability.
  - Arithmetic of fractions.
  - Fraction decimal percent conversion.
  - Order of Operations.
  - Writing algebraic expressions and equations from Guess and Check tables and word problems.
  - Distributive Property and factoring using algebra tiles.
  - Combining like terms.
  - Solving equations using inverse operations with the Cover-up method and with balances and substitution.
  - Solving systems of equations to find the point of intersection.
- Chapters six through 10 cover:
  - Writing and simplifying ratios and proportions.
  - Area of parallelograms, triangles, trapezoid, and circles.
  - Reducing and enlarging figures.
  - Division of fractions.
  - Solving equations with fractional coefficients.
  - Formulas to solve simple and compound interest problems.
  - Markup amounts, selling prices, discounts, sale price and percent of increase or decrease.
  - Distance rate, and time.
  - Writing formulas.
  - Solving literal equations.
  - Pythagorean Theorem.
  - Square roots of square numbers and estimating square numbers.
  - Nets of 2-dimensional drawings for 3-dimensional models.
  - Volume and surface area of cylinders and prisms.
  - Subproblems to break a complex problem into smaller parts.
  - Slopes triangles, rate of change and equations of lines.
  - Line of Best Fit
  - Inequalities
  - Exponential growth.
  - Volume of a cone.
  - Laws of Exponents.
  - Scientific Notation.
- Each chapter includes mental math and reviews the concepts developed previously. All chapters contain a culminating big problem or summary activity to tie together and/or possibly extend the concepts.

## ALGEBRA 1 (CPM 1)—THE BIG PICTURE

- Covers the expected content of any algebra 1 class.
- Reviews over 4-5 chapters what was once traditional algebra content but is now pre-algebra material in some states or offers a six week abridged version allowing students with strong pre-algebra skills to quickly get to chapter six.
- Concepts are based on memorable activities or concrete models so more students will be successful. For example the first experience with graphing equations is an outside activity, the “algebra walk.” Multiplication and factoring of polynomials are introduced with a concrete model, algebra tiles, before moving to the abstract.
- The first five chapters cover:
  - Arithmetic of integers and rational numbers.
  - Graph interpretation and graphing linear and nonlinear functions.
  - Review of plane geometric formulas.
  - Combining like terms and the distributive property.
  - Writing and solving linear equations.
  - Ratios in the context of proportions, similar figures, direct variation, and percent.
- The sixth chapter consolidates what has been covered so far by extending graphing and solving equations to simple systems and extends the distributive property to multiplying binomials.
- Chapters seven through 12 cover:
  - Slope and the equation of lines.
  - Factoring and solving quadratic equations by factoring or the quadratic formula.
  - Using the Pythagorean theorem to focus square roots and using diagrams to write equations.
  - Solving systems by elimination.
  - Properties of exponents.
  - Rational expressions and equations.
  - Functions and absolute value.
  - Problem solving and inequalities.
- The final chapter consolidates and extends using lines and curves of best fit, the proof of the quadratic formula, and more about quadratic functions.
- Each chapter reviews the concepts developed previously and all chapters contain a culminating big problem to tie together and/or possibly extend the algebra concepts.

## GEOMETRY (MATH 2)--THE BIG PICTURE

- Covers the expected content in any geometry course.
- Reviews 85% of Algebra 1 topics explicitly and uses algebra in geometric applications (e.g., supplementary angle problems contain numerical and algebraic expressions as angle measures).
- Focuses logical explanations throughout the text. Provides two and a half chapters specifically focused on proof. Allows as rigorous or relaxed an approach to proof as appropriate for the students.
- The first half of the book introduces the fundamentals of lines, angles, and plane figures (about 70% of the content). The first half of the book:
  - Starts with familiar, concrete topics in an enjoyable format designed for early student success. Topics include the Pythagorean Theorem, area of triangles and quadrilaterals, and linear equations.
  - Introduces proof through logic games and puzzles.
  - Uses the problem solving strategies of organizing data, making tables and lists, and looking for patterns to introduce the concepts for lines and angles.
  - Explores three dimensional visualization, studying prisms and pyramids for the first of two times in the course.
  - Studies transformations and then triangle congruence.
  - The sixth chapter serves as a review of the first half of the course. It introduces several styles of proof and applies them to prove most of the conjectures developed inductively in chapters 0-5.
- The second half of the course concentrates on bigger ideas. Except for circles, each chapter focuses on one or two topics. The second half of the course:
  - Starts with right triangle trigonometry so that students may solve more interesting, complex problems in subsequent chapters.
  - Emphasizes similarity for two dimensional and three dimensional figures.
  - Explores polygons, spending the latter portion of the chapter on proof, including characteristics of quadrilaterals.
  - Introduces the fundamentals of circles, including arcs and angles, followed by the second study of prisms and pyramids, along with cylinders and cones.
  - The last two chapters apply ideas from the course. The first chapter (11) uses area models to study geometric probability. The last chapter offers applications and big problems that use many of the main ideas from the course.
- Constructions are offered in an appendix that can provide an interlude unit of two or three days between the first and second parts of the book.

## ALGEBRA 2 (MATH 3)—THE BIG PICTURE

- Covers the expected content of any algebra 2 class.
- Review and extends 80% of algebra 1 topics and 40% of geometry topics. For example, two dimension systems from algebra 1 are extended to three-dimensional systems and are solved multiple ways. Right triangle trigonometry from geometry is extended to the laws of sines and cosines.
- There is a regular use of technology and a lab approach similar to what is used in science classes.
- Instead of spending the first three to four chapters reviewing algebra 1 as many texts do, the course starts with new material quickly and reviews algebra 1 material in context. For example, chapter one starts with investigating functions using the graphing calculator and chapter two uses arithmetic sequences to review linear equations and systems.
- Eight of the 13 chapters cover the fundamentals of algebra 2:
  - Linear, quadratic, polynomial, exponential, absolute value, simple rational, logarithmic, and square root equations are covered in a functions approach.
  - Conics
  - Sequences and series
  - Systems with and without matrices
  - Complex numbers
  - Whenever possible, topics are investigated in context of real problems. For example, polynomials are concluded with a typical calculus problem of maximum volume.
  - Technology is used to speed investigations and understanding but students are also expected to be able to do most problems by hand.
- The rest of the book gives choices based on expectations of the next course:
  - Two trigonometry chapters—First, a unit circle approach to sine, cosine, tangent functions, and their inverses with applications. Second, right triangle trigonometry is extended to the laws of sines and cosines.
  - Two discrete mathematics chapters—First, a chapter on probability using concrete models to solve probability, conditional probability and expected value problems. Second, a chapter on counting yielding problems involving permutations, combinations, and the binomial theorem.
  - One statistics chapter that covers measures of central tendency and ways of displaying data.
- Each chapter reviews the concepts developed previously and most chapters contain a culminating big problem to tie together and/or possibly extend the algebra concepts

## A SUMMARY OF MATH 4 (Math Analysis)

- This course covers both trigonometry and Pre-Calculus concepts.
  - Approximately 95% of the concepts in a traditional Trigonometry class are covered.
  - The Analysis/Pre-Calculus concepts are similar to other text including limits, vectors, polar and parametric equations, conic sections, matrices, and series.
  - Calculus concepts are studied in more depth than traditional Analysis/Pre-Calculus courses.
- The use of mathematical models is a reoccurring theme throughout the course. The use of statistical concepts is also applied in the use of the models.
  - Unit 1 Introduces models beginning with Linear models and using a median-median line to find a line of best fit. Other models, particularly exponential, are also introduced.
  - Unit 3 and Unit 5 look extensive trigonometric models and applications.
  - Unit 6 uses statistical methods to investigate non-linear data by using regression lines. Use of logarithms to linearize data is also applied.
  - Polar coordinates are investigated between Units 8 and 9 and Unit 12's focus is on parametric equations.
- Concepts of Calculus are investigated with considerably more depth than other Pre-Calculus courses.
  - Unit 2 focus is on area under a curve. Riemann Sums are used to approximate the area under a curve. Methods include left endpoint rectangles, right endpoint rectangles, and trapezoids. Transformations of graphs including piecewise defined functions are also studied.
  - Unit 8 and 9 focus on limits and rates of change. Students look at limits to infinity and at a point, apply concepts of continuity and extend these ideas to include the definition of the derivative. The relationship between rates of change and area under a curve is also explored.
- Additional algebraic techniques that are necessary for calculus and other advanced mathematics courses are explored including rationalization, properties of logarithms, and use of substitution. Extensive understanding of functions and inverses is also developed throughout the course.
- Use of graphing/programmable calculators is extensive. Students will write several programs at different times during the course.
- Units 1-9 are the core units covering all trig concepts and key pre-calculus concepts.
- Unit 10 is designed to teach students to read a college textbook. The material used in on conic sections.
- Units 11-13 as well as Appendix A and B can be chosen from depending on the needs of individual schools. These include matrices, parametric equations, one variable statistics and series.
- Extra problems and skills review are also provided in the appendices.
- Earlier concepts are reviewed and practiced throughout the course.

## A SUMMARY OF CALCULUS

- Covers all content required for the AP Calculus Test – both AB and BC.
- The course starts with five major problems that introduce the big ideas of calculus: optimization, limits, differential equations, exponential functions, the relationship between distance and velocity, piecewise functions, volumes of revolution, volumes by slicing, and the Fundamental Theorem of Calculus. Each of these five major problems is revisited again later in the course for students to solve using new calculus knowledge.
- Each chapter reviews the concepts developed previously and builds on them.
- The curriculum contains several key labs and hands-on activities throughout the course to introduce concepts, such as when students recognize that the rate of a walker relates to the slope of a graph in the "Slope Walk." Labs also develop conceptual understanding, such as when the students discover instantaneous velocity in the "Ramp Lab."
- Students learn about derivatives and integrals simultaneously during the first four chapters and both are presented geometrically and in context.
- The first four chapters cover:
  - Pre-calculus topics, such as trigonometric functions, domain and range, and composite functions
  - Limits and continuity
  - Applications of rates of change, such as velocity and acceleration
  - The difference between average velocity and instantaneous velocity
  - The definition of a derivative and the Power Rule
  - Slope Functions – functions that find the slope of another function for all values in the domain
  - Differentiability and non-differentiability
  - Increasing and decreasing functions and concavity
  - Estimating the area under a curve with a Riemann Sum
  - Area functions – functions that find the area under a curve from 0 to all values in the domain
- The fifth chapter connects derivatives and integrals together with the Fundamental Theorem.
- Chapters six through nine cover:
  - How to find the distance, velocity and acceleration of an object when given information about its position, velocity or acceleration
  - Optimization
  - Related Rates
  - Derivative tools such as the Product, Quotient and Chain Rules, as well as implicit differentiation and finding derivatives of all trigonometric and inverse trigonometric functions
  - The derivative and integral of the natural logarithm and  $y = e^x$

- The Mean Value Theorem
- Integration using Substitution
- Differential Equations and Slope Fields
- Volumes of Revolutions and volumes of known cross-section
- Some material required for the BC Calculus Exam is introduced throughout the course in optional extensions of Chapters 5 - 9. These topics include:
  - Newton's Method
  - l'Hôpital's Rule
  - Improper Integrals, as well as integrating with partial fractions and integrating by parts
  - Arclength
- Chapters ten through thirteen cover additional BC Calculus content, including:
  - Convergence and divergence of infinite series
  - Differentiation and integration of polar functions, as well as parametric functions and vector functions
  - Logistic Curves
  - Approximating functions with polynomials
  - Taylor and Maclaurin Polynomials, as well as the error